

New Unification Theory of Both the All Fundamental Physics Interactions and Noether Theorem

C. Huang¹ and Yong-Chang Huang^{2,3}

¹Department of Physics, Purdue University, 525 Northwestern Avenue,
W. Lafayette, IN 47907-2036, USA

²Institute of Theoretical Physics, Beijing University of Technology,
Beijing 100124, China

³CCAST (World Lab.), P. O. Box 8730, Beijing,
100080, China

April 13, 2017

Abstract

A new unification theory of both the all fundamental physics interactions and Noether theorem is naturally given. The Lagrangians of the well-known fundamental physics interactions are unifiedly deduced from the quantitative causal principle (QCP) and satisfy the gauge invariant principle of general gauge fields interacting with Fermion and/or boson fields. The geometry and physics meanings of gauge invariant property of different physical systems are revealed, and it is discovered that all the Lagrangians of the well-known fundamental physics interactions are composed of the invariant quantities in corresponding spacetime structures. The difficulties that fundamental physics interactions and Noether theorem are not able to be unifiedly given and investigated are overcome, the unified description and origin of the fundamental physics interactions and Noether theorem are shown by QCP, their two-order general Euler-Lagrange Equations and corresponding Noether conservation currents are derived in general curved spacetime. Therefore, using the new unification theory, a lot of research works about different branches of physics etc can be renewedly done and expressed simpler with clear quantitative causal physical meanings.

Key words: Lagrangian, fundamental interaction, unification theory, Noether theorem

PACS: 11.10.-z, 11.10.Ef

1 Introduction

Causal principle should be satisfied in expressing physical laws. In quantum field theory, the causal principle is reflected in the results that if the square of the distance of spacetime coordinates of two boson (or fermion) operators is timelike, the canonical commutator (or anticommutator) of the boson (or fermion) field does not vanish, i.e., their measures are coherent, but not coherent for spacelike case [1]. Dispersion relations are deduced by means of the causal principle etc [2]. Ref. [3] studied the relationships of causal principle and symmetry principle.

It is well known that in physics the well-admitted physical law not satisfying causal principle has not been found. In terms of the condition that causal principle is quantitative, i.e., quantitative causal principle (QCP), Ref. [4] gives a unified theory of all differential variational principles, and a unified theory of all integral variational principles are achieved [5]. The investigations about the generalization from classical statistical mechanics to quantum mechanics still satisfy quantitative causal principle [6]. Refs. [4, 5, 7] shows the quantitative causal principle of gain, loss and transformation of any system, and proves that the equivalent causes must result in the equivalent results, which is just the invariant property of some operations, namely, symmetry properties [8, 9]. In fact, the symmetric properties in physics and mathematics are just the invariant properties under some kinds of operations of systems [9], the relation between symmetries and QCP is given [4, 5], and using QCP, Ref. [8] gives unification theory of different causal algebras and its applications to theoretical physics, in which group theory is included as a special subset.

Utilizing the no-loss-no-gain homeomorphic map transformation satisfying QCP, Ref. [7] overcomes the non-perfect properties of the Volterra process, gains the exact strain tensor formulas in condensed theory, and solves the hard difficulty to give the really serious one-to-one correspondences between the two theories of dislocations in Euclidean space and Weitzenböck Material Manifolds. Ref. [5] gives the unified origin of different variational principle and Noether theorem in terms of QCP. Now it is well known that Noether theorem is deduced from variational principle and group theory [10, 11, 12], while variational principle and Noether theorem may be derived by QCP [5]. It can be seen that this paper and Ref. [4, 5, 7] are essential developments of the investigations of Ref. [3] about the relationships of causal principle and symmetry principle. It is well known that physical fundamental interactions and Noether theorem are used to be regarded as independent fundamental laws being not able to be unifiedly proved and studied up to now, but in the following investigations we show that the two laws are just the deductions of QCP and give the unified discription of the two laws.

In fact, superstring theory has gotten many important developments [13, 14, 15], and Ref. [16] very well investigated Quantum modification of general relativity and so on. But it is well known that there are still some key problems in various kinds of the grand or supersmmetric unification theories up to now, e.g., there are too many adjustable free parameters etc in various kinds of the grand or supersmmetric unification theories, which are not natural, even superstring

theories and M-theory have themselves unsolved key problems, e.g., we still cannot seriously break the high dimensional theories down to four dimensions and do rigorous phenomenology etc. Therefore, exploring the other possible unification theories is actually needed, which may activate to understand and solve all the problems finally, that is, we may try to solve all the problems from different aspects, which may finally activate development and perfection of the various kinds of the unification theories, e.g., superstring theories, M-theory and so on.

The arrangement of this Letter is: Sect. 2 is a new unification theory of fundamental physics interactions, Sect. 3 is the unified description of fundamental physics interactions and Noether theorem, Sect. 4 is applications of this theory, and Sect.5 is summary and conclusion.

2 A New Unification Theory of fundamental physics Interactions

For the convenience of research, we simply review the proof of QCP. In physics, quantitative action (cause) of some quantities must lead to the equal action (result), i.e., how much lose (cause), how much gain (result), which is just the QCP deduced from the no-loss-no-gain principle in the universe [4, 5] and it may be concretely expressed as

$$DS - CS = 0 \quad (1)$$

Eq.(1) means that any quantitative action produced by operator set D acting on S must lead to appearance of set C acting on S so that DS equates CS, where D and C may be different operator sets, the whole process satisfies the QCP so that right hand side of Eq.(1) keeps zero, i.e. satisfies the no-loss-no-gain principle in the universe [5]. Thus, Eq.(1) is viewed as a mathematical expression of the QCP. Eq.(1) is very useful for the following studies. Eq.(1)'s nonlinear expression refers to Ref. [8].

For a 4-dimensional physical general curved spacetime M^4 (*there always naturally exist the constructions of vector bundle $E(M^4, F, \pi, G)$ or associate vector bundle $E(M^4, F, \pi_V, G, P(M^4, G, \pi))$ of principal bundle $P(M^4, G, \pi)$. Readers, not familiar with manifold and fibre bundle, don't need to read the Italic type parts in parenthesis, which will not affect their understanding on the paper, as the same below*), taking S_V as a general basic vector field in any open neighborhood U_V in M^4 , D as differential connection operator, C as connection ω_V in Eq.(1), it follows that [17, 18]

$$DS_V = \omega_V S_V \quad (2)$$

Eq.(2)'s physical meaning is that the quantitative physical effect operator D's acting on S_V must equate that the gauge field or connection ω_V times S_V (*in which S_V may be a basic vector of principal bundle whose base manifold is M^4 or a basic vector on M^4 of vector bundle*).

Substituting the transformation $S_V = A_{VU}S_U$ into Eq.(2), it follows that [17]

$$DS_V = dA_{VU}S_U + A_{VU}\omega_U S_U = \omega_V S_V \quad (3)$$

where $DA_{VU} = dA_{VU}$, because A_{VU} is the matrix function transforming S_U into S_V between two open neighborhoods U_U and U_V ($U_U \cap U_V \neq 0$) on M^4 .

Substituting $A_{VU}^{-1}S_V = S_U$ into Eq.(3), we obtain the relation transforming connection ω_U into connection ω_V as follows [17]

$$\omega_V = dA_{VU}A_{VU}^{-1} + A_{VU}\omega_U A_{VU}^{-1} \quad (4)$$

For gauge field (*in principal bundle*), it is just gauge transformation between gauge fields ω_U and ω_V in two open neighborhoods U_U and U_V ($U_U \cap U_V \neq 0$) in gauge field theory; for connection (*in vector bundle*), it is the transferring relation between connections ω_U and ω_V in two open neighborhoods U_U and U_V ($U_U \cap U_V \neq 0$) in curved spacetime, i.e., Eq.(4) is just the unified transformation expression of gauge field and connection in two open neighborhoods U_U and U_V , thus we can generally call the unified transformation as general gauge transformation.

Using Eq.(4), we have $\omega_V A_{VU} - A_{VU}\omega_U = dA_{VU}$, and further acting the differential connection operator D on Eq.(3), we obtain not only the unification expression of curvature tensor 2-form (*in tangent vector bundle*) and field strength tensor 2-form (*in principal bundle*) but also the unification expression of general coordinate transformation of curvature tensor 2-form (*in tangent vector bundle*) and gauge transformation of field strength tensor 2-form (*in principal bundle*) as follows

$$\Omega_V = A_{VU}\Omega_U A_{VU}^{-1} = d\omega_V - \omega_V \wedge \omega_V \quad (5)$$

In fact, Eqs.(4&5) show that gravity is also a kind of gauge theory.

Multiplying Eq.(5)'s component quantity expression of the curvature tensor 2-form (*in tangent vector bundle*)

$$\frac{1}{2}\Omega_{aij}^b dx^i \wedge dx^j = \frac{1}{2}A_a^{b'} \Omega_{b'i'j'}^{c'} A_{c'}^{-1b} dx^{i'} \wedge dx^{j'} \quad (6)$$

with $g^{ak}\varepsilon_{bklm}dx^l \wedge dx^m$ from right hand side, in which the $a, b, c, \dots, i, j, k, \dots$ and $a', b', c', \dots, i', j', k', \dots$ are in U_V and U_U respectively, defining $\varepsilon_{1234} = (-g)^{\frac{1}{2}}$ [19] & $x^4 = ict$ and using $\varepsilon_{i_1 i_2 i_3 i_4} \varepsilon^{j_1 j_2 j_3 j_4} = \text{sgn}(g) \delta_{i_1 i_2 i_3 i_4}^{j_1 j_2 j_3 j_4}$ [19], we prove

$$\Omega_{ab}^{ba}(-g)^{\frac{1}{2}} dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 = \Omega_{a'b'}^{b'a'}(-g')^{\frac{1}{2}} dx'^1 \wedge dx'^2 \wedge dx'^3 \wedge dx'^4 \quad (7)$$

Namely, Eq.(7) is invariant or doesn't depend on coordinates of different U_V and U_U .

Adding integral sign to Eq.(7) and, as usual, defining $\Omega_{ab}^{ba} = g^{bc}R_{cab}^a = R_V, \Omega_{a'b'}^{b'a'} = R_U$ and $d\tau = dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4$, we obtain the invariant gravitational action [20]

$$A = \int_{U_U \cap U_V} R_U (-g)^{\frac{1}{2}} d\tau_U = \int_{U_U \cap U_V} R_V (-g)^{\frac{1}{2}} d\tau_V \quad (8)$$

Thus, Eq.(8) keeps invariant in the whole spacetime (manifold $M^4 = \bigcup_V U_V$), because the two open neighborhoods U_U and U_V are two arbitrary open overlapped neighborhoods in the global spacetime (*manifold in the vector bundle*). The above condition (8) is just the condition that Eq.(8) may be taken as invariant gravitational action in the whole spacetime, because the physical consistence demands that Eq.(8) must take the invariant formulation in every open neighborhood of the whole spacetime, which satisfies just the general gauge invariant principle of gravitational gauge fields. Then geometry and physics meanings of gauge invariant property are directly, whole and seriously revealed.

A Noether invariant quantity is obtained from Eq.(5) in terms of 2-form field strength tensor (*in principal bundle*) as follows $tr(\Omega_V \wedge \Omega_V) = tr(\Omega_U \wedge \Omega_U)$, which satisfies just the general gauge invariant principle [21] of general gauge fields. Then geometry and physics meanings of gauge invariant property of general gauge fields are directly, whole and seriously revealed.

Adding integral sign to $tr(\Omega_V \wedge \Omega_V)$ and using $tr(T^a T^b) = -2\delta^{ab}$, we have the invariant action with dual gauge field strength tensor in the global curved spacetime as follows [20]

$$A = \int \frac{1}{2\pi^2 \kappa} tr(\Omega \wedge \Omega) = - \int \frac{1}{2\pi^2 \kappa} \Omega_{ij}^a \Omega^{aij} (-g)^{\frac{1}{2}} d\tau \quad (9)$$

Thus the Lagrangian density is $\mathcal{L} = -\frac{(-g)^{\frac{1}{2}}}{2\pi^2 \kappa} \Omega_{ij}^a \Omega^{aij} = -\frac{(-g)^{\frac{1}{2}}}{4q^2} \Omega_{ij}^a \Omega^{aij}$ ($\Omega^{aij} = \frac{1}{2} \varepsilon^{ijkl} \Omega_{kl}^a$ is dual gauge field strength tensor, taking $\kappa = 2q^2/\pi^2$) with dual gauge field strength tensor existing in the global spacetime [18, 20]. As Eq.(9)'s natural generalization, for higher dimensions, we may naturally deduce $A = \int \frac{1}{2\pi^2 \kappa} tr(\Omega \wedge \Omega \wedge \Omega \dots \wedge \Omega)$.

In terms of modern differential geometry, we can define the inner product $\langle \partial_\mu, \partial_\nu \rangle = \langle e_\mu, e_\nu \rangle = g_{\mu\nu}$ of natural (or called intrinsical) tangent basic vectors and the inner product $\langle dx^\mu, dx^\nu \rangle = \langle e^\mu, e^\nu \rangle = g^{\mu\nu}$ of natural (or called intrinsical) cotangent basic vectors, they are consistent, see Appendix A, then we obtain an important invariant $\langle \omega_\mu dx^\mu, \omega_\nu dx^\nu \rangle = \omega_\mu \omega_\nu g^{\mu\nu} = \omega_\mu \omega^\mu$ of inner product of two 1-forms, i.e., cotangent vectors, under coordinate transformation. Similar to deducing the important invariant $\omega_\mu \omega^\mu$ of inner product of two 1-forms, see Appendix B we finally achieve the invariant action of general gauge fields in the global curved spacetime as follows

$$A = \int \frac{1}{2\pi^2 \kappa} tr \langle \Omega, \Omega \rangle (-g)^{\frac{1}{2}} d\tau = \int \frac{1}{4\pi^2 \kappa} \Omega_{ij}^a \Omega^{aij} (-g)^{\frac{1}{2}} d\tau \quad (10)$$

Eq.(10) satisfies just the general gauge invariant principle in the global spacetime.

When S_{U_i} ($i = 1, 2, 3, \dots, k$; U is subscript of the open neighborhood U_U) are k linearly independent basic vectors of the vector space (of vector bundle or associate vector bundle $E(M^4, F, \pi_V, G, P(M^4, G, \pi))$), or equivalently the orthonormal basic vectors of the Group G 's representation vector space F , Therefore, any vector field S may be expressed as

$$S = S_U \Psi_U \quad (11)$$

where Ψ_U is complex column matrix function of the corresponding components. It follows from Eq.(2) that

$$D' S_{U_i} = \omega'_{U_{ij}} S_j = -S_j \omega'_{U_{ji}} \quad (12)$$

in which we have remarked

$$D' = \gamma^\mu D_\mu, \quad d' = \gamma^\mu \partial_\mu, \quad \omega'_{U_{ij}} = \gamma^\mu \omega_{U_{\mu ij}} \quad (13)$$

where γ^μ is γ matrix with Lorentz superscript μ in Eq.(13), and $dx'^\mu = a^\mu_\nu dx^\nu$ ($\mu, \nu = 0, 1, 2, 3$) have the same transformation law as $\gamma'^\mu = a^\mu_\nu \gamma^\nu$ [22], a^μ_ν is Lorentz group matrix element. Because the i, j, \dots and μ, ν, \dots belong to the different freedoms' groups, thus, their matrixes corresponding to different groups are commutable. From Eqs.(2), (11) and (12) we have

$$D' S = \omega'_U S_U \Psi_U + S_U d' \Psi_U = S_U (d' \Psi_U - \omega'_U \Psi_U) \quad (14)$$

Using the orthonormal relation (we can choose the orthonormal basic vectors of the vector space)

$$(\bar{S}_{U_i}, S_{U_j}) = \delta_{ij} \quad (15)$$

we obtain

$$(\bar{S}, D' S) = (\bar{\Psi}_U \bar{S}_U, S_U (d' \Psi_U - \omega'_U \Psi_U)) = \bar{\Psi}_U (d' \Psi_U - \omega'_U \Psi_U) \quad (16)$$

Now consider the expression of Eq.(16) in an open neighborhood U_W . It follows from $\Psi_U = A_{UW} \Psi_W$ that

$$d' \Psi_U = d' A_{UW} \Psi_W + A_{UW} d' \Psi_W \quad (17)$$

Using Eqs.(4), (16), (17) and $\Psi_U = A_{UW} \Psi_W$ (A_{UW} is transformation matrix), we have

$$\begin{aligned} \bar{\Psi}_U (d' \Psi_U - \omega'_U \Psi_U) &= \bar{\Psi}_W A_{UW}^+ [d' A_{UW} \Psi_W + A_{UW} d' \Psi_W \\ &\quad - (d' A_{UW} A_{UW}^{-1} + A_{UW} \omega'_W A_{UW}^{-1}) A_{UW} \Psi_W] = \bar{\Psi}_W (d' \Psi_W - \omega'_W \Psi_W) \end{aligned} \quad (18)$$

where we have used $A_{UW}^+ A_{UW} = I$. Namely Eq.(18) is the invariant quantity in the whole spacetime (*manifold in the bundle*), and the physical consistence demands that Eq.(18) must take the invariant formulation in the every open neighborhood of the whole spacetime, which satisfies just the general gauge invariant principle of the gauge fields interacting with Fermi fields. Then geometry and physics meanings of gauge invariant property of gauge fields interacting with Fermion fields are directly, whole and seriously revealed.

Inserting Eq.(13) into Eq.(18), we obtain the general topological invariant Lagrangians of matter field Ψ interacting with gauge field ω_μ , which keeps effective in the whole spacetime as follows

$$\mathcal{L}_\Psi = \bar{\Psi} \gamma^\mu (\partial_\mu \Psi - \omega_\mu \Psi) \quad (19)$$

Now we consider another invariant.

Similar to the discussion of Eqs.(14&18), taking $D = dx^\mu D_\mu$, $d = dx^\mu \partial_\mu$, $\omega_{Uij} = dx^\mu \omega_{U\mu ij}$, replacing spinor field Ψ_U with scalar field φ in Eq.(14), it follows that $DS = S_U(d\Psi_U - \omega_U \Psi_U)$, further taking Hermite conjugation of DS , and making an inner product, relevant to $(\bar{S}_{Ui}, S_{Uj}) = \delta_{ij}$ and $\langle dx^\mu, dx^\nu \rangle$, of both DS and its conjugation, we get

$$\begin{aligned} \langle \overline{DS}, DS \rangle &= \langle (\bar{\varphi}_U(\bar{d} - \bar{\omega}_U) \bar{S}_U, S_U(d\varphi_U - \omega_U \varphi_U)) \rangle \\ &= \langle \bar{\varphi}_U(\bar{\partial}_\mu - \bar{\omega}_{\mu U}) dx^\mu, dx^\nu (\partial_\nu \varphi_U - \omega_{\nu U} \varphi_U) \rangle \end{aligned} \quad (20)$$

Analogous to the discussion of Eq.(18), it is easy to prove that Eq.(20) is the invariant quantity existing in the whole spacetime.

In terms of modern differential geometry, we again use the inner product $\langle dx^\mu, dx^\nu \rangle = \langle e^\mu, e^\nu \rangle = g^{\mu\nu}$ of natural cotangent basic vectors, then we achieve an important invariant Lagrangian of scalar fields interacting with gauge fields

$$\begin{aligned} \mathcal{L}_\varphi &= \langle \overline{DS}, DS \rangle = \bar{\varphi}(\bar{\partial}_\mu - \bar{\omega}_\mu) g^{\mu\nu} (\partial_\nu - \omega_\nu) \varphi \\ &= \bar{\varphi}(\bar{\partial}_\mu - \bar{\omega}_\mu)(\partial^\mu - \omega^\mu) \varphi \end{aligned} \quad (21)$$

Thus, Eq.(21) satisfies the physical invariant consistent demand in the every open neighborhood of the global spacetime. That is, Eq.(21) satisfies just the general gauge invariant principle of the gauge fields interacting with scalar fields. Then geometry and physics meanings of gauge invariant property of gauge fields interacting with Boson fields are directly, whole and seriously revealed.

On the other hand, due to

$$(\bar{S}, S) = (\bar{\varphi}_U \bar{S}_U, S_U \varphi_U) = \bar{\varphi}_U \varphi_U \quad (22)$$

and using $\varphi_U = A_{UW} \varphi_W$, it is easy to prove that Eq.(22) is invariant quantity existing in the whole spacetime. Thus we may multiply Eq.(22) with mass

parameter m^n ($n = 2$, taking φ as boson function; $n = 1$, taking φ as fermion function) to construct the mass part of the Lagrangian of a system.

$$\mathcal{L}_{m\varphi} = m^n \bar{\varphi} \varphi, \quad (23)$$

About potential functional of $(\bar{\varphi}_U, \varphi_U)$ due to the invariance of $(\bar{\varphi}_U, \varphi_U)$ under the general gauge transformation, it must satisfy

$$V_U(\bar{\varphi}_U, \varphi_U) = V_W(\bar{\varphi}_W, \varphi_W) \quad (24)$$

which is the invariant condition in the global spacetime, i.e., Eq.(24) satisfies just the general gauge invariant principle of potential energy. For example, Eq.(22)'s arbitrary combinations may satisfy condition (24)., i.e., we may generally take Eq.(22) as the variable to construct scalar potential functional. e. g., the potential functional V of $SU(2)$ complex scalar fields

$$V(\bar{\varphi}\varphi) = \lambda^2(\bar{\varphi}\varphi - \mu^2)^2. \quad (25)$$

Using Eqs.(8), (10), (19), (21). (23) and (24), we achieve gravitational action, all general actions of matter fields, spinor fields and scalar fields interacting with gauge fields in the curved spacetime as follows

$$A = \int_{M^4} (\alpha R + \mathcal{L}_m)(-g)^{\frac{1}{2}} d\tau \quad (26)$$

$$A = \int_{M^4} [\alpha R + \bar{\Psi} \gamma^\mu (\partial_\mu \Psi - \omega_\mu \Psi) + m \bar{\Psi} \Psi + U(\bar{\Psi}, \Psi) - \frac{1}{4q_\lambda^2} F_{\mu\nu}^{\lambda a} F^{\lambda a \mu\nu}] (-g)^{\frac{1}{2}} d\tau \quad (27)$$

$$A = \int_{M^4} [\alpha R + \bar{\varphi} (\partial^\mu - \bar{\omega}^\mu) (\partial_\mu - \omega_\mu) \varphi + m^2 \bar{\varphi} \varphi + V(\bar{\varphi}, \varphi) - \frac{1}{4q_\lambda^2} F_{\mu\nu}^{\lambda a} F^{\lambda a \mu\nu}] (-g)^{\frac{1}{2}} d\tau \quad (28)$$

where α is a parameter, \mathcal{L}_m is the Lagrangian of general matter fields, $\bar{\omega}^\mu = g^{\mu\nu} \bar{\omega}_\nu$ and $\gamma^\mu = g^{\mu\nu} \gamma_\nu$. (In the case that the fibre G of principal bundle is a semisimple group of a general form,) the Lagrangian contains r arbitrary constants q_λ , $\lambda = 1, 2, \dots, r$, in which r is the number of invariant simple factors [20].

The fundamental physics interactions, e.g., strong, weak, electromagnetic and gravitational interactions, can be described by Eqs.(26-27), these actions are unifiedly deduced by QCP, i.e., Eq.(1), thus the new unification theory of fundamental physics interactions is given in terms of modern differential geometry. The more concrete examples are that the known grand unification theories may be $SU(5)$, $S(10)$ or E_6 gauge theories, and it is very easy that their relative more fermion mass terms etc can similarly be deduced by using the expression which is relevant to spinor's general gauge invariant quantities.

3 The Unified Description of fundamental physics Interactions and Noether Theorem

In the expression (1) of the QCP, which has deduced all the physical fundamental interactions in this paper, for general field variables $X(x) = \{\Psi(x), \varphi(x), \omega_\mu(x), g_{\mu\nu}(x), \dots\}$ above, when S is the actions (26-28), C is unit element and D is infinitesimal transformation operator of continuous Lie group [5, 11, 12]

$$x^\mu \rightarrow x'^\mu \doteq x^\mu + \Delta x^\mu = x^\mu + \varepsilon_\sigma \tau^{\mu\sigma}(x, X, X_{,\mu}) \quad (29)$$

$$X^a(x) \rightarrow X'^a(x') \doteq X^a(x) + \Delta X^a(x) = X^a(x) + \varepsilon_\sigma \xi^{a\sigma}(x, X, X_{,\mu}) \quad (30)$$

in which ε_σ ($\sigma = 1, 2, \dots, m$) are infinitesimal parameters of Lie group D_m in Eq.(1). Thus, we get the unified expression of their variational principles, as follows

$$\Delta A = DA - A = A' - A = 0 \quad (31)$$

Not losing generality, under the transformations of Eqs.(29) and (30), we have

$$L'(x', X'(x'), X'_{,\mu}(x'), X'_{,\mu\nu}(x')) = L(x', X'(x'), X'_{,\mu}(x'), X'_{,\mu\nu}(x')) + \partial_\mu \Omega^\mu \quad (32)$$

where $\partial_\mu \Omega^\mu = \varepsilon_\sigma \partial_\mu \Omega^{\sigma\mu}$ and the Ricci Scalar R contains $g_{\alpha\beta, \mu\nu}$. Using the unified expression of the variational principles, we have

$$\begin{aligned} \Delta A = \int_{M^4} \{ & [\frac{\partial L}{\partial X^a} - \partial_\mu \frac{\partial L}{\partial X^a_{,\mu}} + \partial_\mu \partial_\nu \frac{\partial L}{\partial X^a_{,\mu\nu}}] \delta X^a + \\ & \partial_\mu [L \Delta x^\mu + (\frac{\partial L}{\partial X^a_{,\mu}} - \partial_\nu \frac{\partial L}{\partial X^a_{,\mu\nu}}) \delta X^a + \frac{\partial L}{\partial X^a_{,\mu\nu}} \delta X^a_{,\nu} + \Omega^\mu] \} d^4x \end{aligned} \quad (33)$$

in which $\delta X^a = \Delta X^a - X^a_{,\nu} \Delta x^\nu$. Using the above discussions, we can obtain their two-order general Euler-Lagrange Equations and corresponding Noether conservation currents as follows

$$\frac{\partial L}{\partial X^a} - \partial_\mu \frac{\partial L}{\partial X^a_{,\mu}} + \partial_\mu \partial_\nu \frac{\partial L}{\partial X^a_{,\mu\nu}} = 0 \quad (34)$$

$$\partial_\mu J^{\mu\sigma} = 0 \quad (35)$$

$$J^{\mu\sigma} = L \tau^{\mu\sigma} + (\frac{\partial L}{\partial X^a_{,\mu}} - \partial_\nu \frac{\partial L}{\partial X^a_{,\mu\nu}}) (\xi^{a\sigma} - X^a_{,\alpha} \tau^{\alpha\sigma}) + \frac{\partial L}{\partial X^a_{,\mu\nu}} \partial_\nu (\xi^{a\sigma} - X^a_{,\alpha} \tau^{\alpha\sigma}) + \Omega^{\mu\sigma} \quad (36)$$

Therefore, Noether theorem of the general physical system is deduced by QCP.

In all, using QCP, i.e., Eq.(1), we derive not only all the well-known fundamental physics Lagrangians and their variational principles and further Noether

theorem, but also their two-order general Euler-Lagrange Equations and corresponding Noether conservation currents. Applying the above studies, variation laws of the different physical systems are naturally determined. Therefore, it is essential to deduce effectively unified expressions of elementary physical laws by QCP.

4 Applications

We can use the new unification theory to different physical systems. For example:

(1) As Eq.(24)'s natural generalization, when the invariant scalar product $\overline{\varphi}\varphi$ is extended by the invariant scalar curvature R , we may naturally deduce $f(R, \overline{\varphi}\varphi)$ gravity, i.e., $A = \int f(R, \overline{\varphi}\varphi)(-g)^{\frac{1}{2}}d\tau$, because $(-g)^{\frac{1}{2}}d\tau$ is invariant volume element. Further adding the first term αR of Eq.(28), we get $A_1 = \int F(R, \overline{\varphi}\varphi)(-g)^{\frac{1}{2}}d\tau = \int [f(R, \overline{\varphi}\varphi) + \alpha R](-g)^{\frac{1}{2}}d\tau$ [23].

(2) As Eq.(24)'s direct application, when the invariant scalar product $\overline{\varphi}\varphi$ is taken as the invariant scalar product H^+H of Higgs field H's $\{5\}$ multiplets, we may naturally deduce $V(H) = \frac{\mu_0^2}{2}H^+H + \frac{\lambda}{4}(H^+H)^2$ (μ_0 and λ are parameters), which is just the potential of Higgs field H's $\{5\}$ multiplets for grand unification theory [24]; for Higgs field ϕ 's $\{24\}$ multiplets that are expressed by a 5×5 matrix, the matrix satisfies Eq.(5)'s general gauge invariant property, i.e., $\phi_V = A_{VU}\phi_U A_{VU}^{-1}$ or $tr\phi_V = tr\phi_U$ and more generally it follows that $tr\phi_V^n = tr\phi_U^n$, and as Eq.(24)'s natural generalization, we have $V(\phi) = -\frac{1}{2}\mu^2 tr\phi^2 + \frac{a}{4}(tr\phi^2)^2 + \frac{b}{2}tr\phi^4$ (μ , a and b are parameters); for mixed invariants of H's $\{5\}$ and ϕ 's $\{24\}$ multiplets, similarly people can deduce $V(H, \phi) = cH^+H(tr\phi^2) + dH^+\phi^2H$ (c and d are parameters), thus we totally deduce $V_t(H, \phi) = V(H) + V(\phi) + V(H, \phi)$, which is the total Higgs potential of grand unification theory and the same as that of Ref. [24].

(3) Generalizing R as an invariant functional $f(R)$ and using Eq.(27) deduced by QCP, people can deduce Fermion field Lagrangian interacting with Non-Abelian gauge field and general $f(R)$ gravitational field as follows

$$A = \int_{M^4} [\alpha f(R) + \overline{\Psi}[\gamma^\mu(\partial_\mu \Psi - \omega_\mu \Psi) + m]\Psi + U(\overline{\Psi}, \Psi) - \frac{1}{4q_\lambda^2} F_{\mu\nu}^{\lambda a} F^{\lambda a \mu\nu}](-g)^{\frac{1}{2}}d\tau. \quad (37)$$

Eq.(37) is just a general generalization of Eq.(27), and further utilizing the deduced Eqs.(34) and (35) by QCP, people can naturally give the new unification theory of the fundamental Non-Abelian gauge field, Fermion field and general $f(R)$ gravitational field interactions and Noether theorem by using QCP, and can concretely derive their Euler-Lagrange Equations and corresponding Noether conservation currents as done in Sect. 3. Thus the relative books and articles may be renewedly and systematically rewritten, which will help people to understand and express the fundamental Non-Abelian gauge field, Fermion

field and general $f(R)$ gravitational field interactions simpler and with clear quantitative causal physical meanings.

(4) Generalizing R as an invariant functional $f(R)$ and using Eq.(28) deduced by QCP, people can deduce Boson field Lagrangian interacting with Non-Abelian gauge field and general $f(R)$ gravitational field as follows

$$A = \int_{M^4} [\alpha f(R) + \bar{\varphi}(\bar{\partial}^\mu - \bar{\omega}^\mu)(\partial_\mu - \omega_\mu)\varphi + m_\varphi^2 \bar{\varphi}\varphi + V(\bar{\varphi}, \varphi) - \frac{1}{4q_\lambda^2} F_{\mu\nu}^{\lambda a} F^{\lambda a \mu\nu}] (-g)^{\frac{1}{2}} d\tau. \quad (38)$$

Eq.(38) is just a general generalization of Eq.(28), and further utilizing the deduced Eqs(34) and (35) by using QCP, people can naturally give new unification theory of the fundamental Non-Abelian gauge field and general $f(R)$ gravitational field interactions and Noether theorem by using QCP, and can concretely derive their Euler-Lagrange Equations and corresponding Noether conservation currents. Thus the relevant books and articles may be renewedly and systematically rewritten, which will help people to understand and express the fundamental Non-Abelian gauge field, Boson field and general $f(R)$ gravitational interactions simpler and with clear quantitative causal physical meanings. When $f(R) = 0$, we naturally give Electric and color superconduction theories corresponding Abelian $-\frac{1}{4q^2} F_{\mu\nu} F^{\mu\nu}$ and non-Abelian $-\frac{1}{4q_\lambda^2} F_{\mu\nu}^{\lambda a} F^{\lambda a \mu\nu}$ in Eq.(38), respectively [24], thus the relevant books and articles may be renewedly and systematically rewritten, which will help people to understand and express the fundamental Non-Abelian gauge field and Boson field interactions simpler and with clear quantitative causal physical meanings.

(5) Using QCP, people can more generally deduce

$$A = \int_{M^4} [\alpha f(R) + \bar{\Psi}\gamma^\mu(\partial_\mu\Psi - \omega_\mu\Psi) + m_\Psi\bar{\Psi}\Psi + \bar{\varphi}(\bar{\partial}^\mu - \bar{\omega}^\mu)(\partial_\mu - \omega_\mu)\varphi + m_\varphi^2\bar{\varphi}\varphi + M(U(\bar{\Psi}, \Psi), V(\bar{\varphi}, \varphi)) + V_t(H, \phi) - \frac{1}{4q_\lambda^2} F_{\mu\nu}^{\lambda a} F^{\lambda a \mu\nu}] (-g)^{\frac{1}{2}} d\tau. \quad (39)$$

Eq.(39) is just a general generalization of Eqs.(37) and (38), and further utilizing the deduced Eqs(34) and (35) by QCP, people can naturally give the new unification theory of the fundamental Non-Abelian gauge field, Fermion field, Boson field, general $f(R)$ gravitational field interactions and Noether theorem by using QCP, and can concretely derive their Euler-Lagrange Equations and corresponding Noether conservation currents. Thus the relevant books and articles may be renewedly and systematically rewritten, which will make people understand and express the fundamental Non-Abelian gauge field, Fermion field, Boson field, and general $f(R)$ gravitational field interactions simpler and with clear quantitative causal physical meanings. Specially, Eq.(39) is just the action of the theory of unifiedly describing all known physical fundamental interactions, further further utilizing the deduced Eqs(34) and (35) by QCP, consequently the new unification theory of the fundamental physics interactions and Noether theorem is given by using QCP for the first time.

Because of length limitation of the Letter, the other Lagrangians in physics may be analogously derived, their detailed applications can be given easily by means of the research of this paper and which will be written in our following papers.

5 Summary and Conclusion

This paper naturally gives the new unification theory of both all the fundamental physics interactions and Noether theorem by utilizing QCP, i.e., Eq.(1). The invariant quantities covering all the parts of the Lagrangians of the well-known fundamental physics interactions are deduced by QCP, all these invariant quantities satisfy general gauge invariance of the general gauge fields interacting with Fermion and/or boson fields. These different invariant quantities are constructed (*in vector bundle or associate vector bundle of principal bundle, whose base manifolds are a 4-dimensional physical general curved spacetime manifold M^4*). Thus, using these invariant quantities, all the Lagrangians of the well-known physical fundamental interactions are given and satisfy the general gauge invariant principle of general gauge fields interacting with Fermion and/or boson fields, and geometry and physics meanings of gauge invariant property of different physical systems are directly, whole and seriously revealed.

Actually it is in terms of QCP that their variational principles and corresponding Noether theorem in field theory are derived. Therefore, the very hard difficulty (that the fundamental physics interactions and variational principle, further Noether theorem, previously regarded as independent fundamental laws, were not able to be unifiedly proved and investigated in the past [25]) is naturally overcome in this Letter. Namely, the unified description and origin of fundamental physics interactions and the Noether theorems is shown by QCP, furthermore, their two-order general Euler-Lagrange Equations and corresponding Noether conservation currents are obtained.

In fact, general physics process [4, 5, 8] and different physics processes [26, 27, 28, 29] all satisfy QCP with no-loss-no-gain character, the above investigations satisfy QCP, and are consistent. Especially, using the theory of this Letter, a lot of research works about different branches of physics etc can be renewedly done and expressed simpler with clear quantitative causal physical meanings, e.g., people can deduce a lot of different parts of different Lagrangians, which will naturally give Lagrangians of different physics systems according to QCP and different symmetries etc, and then using Eqs(34-36) people can give all corresponding concrete physics laws, i.e., find new Euler-Lagrange Equations and conservation laws, and discover new concrete physics laws and so on. Thus the theory of this Letter is useful and will be broad utilized and cited in different branches of physics and so on. Therefore, all articles and books relevant to the fundamental physics interactions and Noether theorem may be supplemented with the new conclusions and this Letter will be cited by using the new unification theory in different physical systems, e.g., condensed physics, atomic

physics, molecular physics, quantum optics, nuclear physics, particle physics and so on.

Acknowledgments

Authors are grateful to Prof. Z. P. Li for useful discussion and comment.

The work is supported by NSF through grants PHY-0805948, DOE USA through grant de-sc0007884, and National Natural Science Foundation of China (No. 11275017 and No. 11173028).

Appendix A

For any local natural coordinate x^μ and local orthogonal coordinate x^a in any 4-dimensional general spacetime, any tangent vector along any curve line t is $X = \frac{d}{dt} = \frac{dx^\mu}{dt} \frac{\partial}{\partial x^\mu} = \frac{dx^a}{dt} \frac{\partial}{\partial x^a}$, then the inner product of the tangent vector equates

$$\langle X, X \rangle = \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} \langle \frac{\partial}{\partial x^\mu}, \frac{\partial}{\partial x^\nu} \rangle = \frac{dx^a}{dt} \frac{dx^b}{dt} \langle \frac{\partial}{\partial x^a}, \frac{\partial}{\partial x^b} \rangle \quad (\text{A1})$$

where we may define the inner product $\langle \partial_\mu, \partial_\nu \rangle = \langle e_\mu, e_\nu \rangle = g_{\mu\nu}$ (i.e., natural covariant metric) of natural tangent basic vectors and the inner product $\langle \partial_a, \partial_b \rangle = \langle e_a, e_b \rangle = \eta_{ab}$ ($\eta_{ab} = -1$, as $a = b = 0$; $\eta_{ab} = 1$, as $a = b = 1, 2, 3$; $\eta_{ab} = 0$, as $a \neq b$, i.e., orthonormal covariant metric) of orthonormal tangent basic vectors, so we can deduce the infinitesimal line element square

$$ds^2 = \langle X, X \rangle (dt)^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ab} dx^a dx^b \quad (\text{A2})$$

Similar to $\langle \partial_\mu, \partial_\nu \rangle = g_{\mu\nu}$ and $\langle \partial_a, \partial_b \rangle = \eta_{ab}$, we define the inner product of natural (or called intrinsical) cotangent basic vectors as $\langle dx^\mu, dx^\nu \rangle = \langle e^\mu, e^\nu \rangle = g^{\mu\nu}$ (i.e., natural contravariant metric) and the inner product of orthonormal cotangent basic vectors as $\langle dx^a, dx^b \rangle = \langle e^a, e^b \rangle = \eta^{ab}$ ($\eta^{ab} = -1$, as $a = b = 0$; $\eta^{ab} = 1$, as $a = b = 1, 2, 3$; $\eta^{ab} = 0$, as $a \neq b$, i.e., orthonormal contravariant metric), thus, we prove

$$\begin{aligned} g_{\mu\nu} g^{\mu\alpha} &= \langle \frac{\partial}{\partial x^\mu}, \frac{\partial}{\partial x^\nu} \rangle \langle dx^\mu, dx^\alpha \rangle \\ &= \frac{\partial x^a}{\partial x^\mu} \frac{\partial x^b}{\partial x^\nu} \langle \frac{\partial}{\partial x^a}, \frac{\partial}{\partial x^b} \rangle \frac{dx^\mu}{dx^c} \frac{dx^\alpha}{dx^d} \langle dx^c, dx^d \rangle \\ &= \frac{\partial x^a}{\partial x^c} \frac{\partial x^b}{\partial x^d} \eta_{ab} \frac{dx^\alpha}{dx^d} \eta^{cd} = \delta_c^a \frac{\partial x^b}{\partial x^d} \eta_{ab} \frac{dx^\alpha}{dx^d} \eta^{cd} = \delta_\nu^\alpha \end{aligned} \quad (\text{A3})$$

Furthermore, using Eq.(A2) we deduce $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ab} \frac{\partial x^a}{\partial x^\mu} \frac{\partial x^b}{\partial x^\nu} dx^\mu dx^\nu = \eta_{ab} e_\mu^a e_\nu^b dx^\mu dx^\nu$, i.e., we prove

$$g_{\mu\nu} = \eta_{ab} \frac{\partial x^a}{\partial x^\mu} \frac{\partial x^b}{\partial x^\nu} = \eta_{ab} e_\mu^a e_\nu^b \quad (\text{A4})$$

where $\frac{\partial x^a}{\partial x^\mu} = e_\mu^a$ is vierbein. On the other hand, we may directly prove Eq.(A4) by using $g_{\mu\nu} = \langle \frac{\partial}{\partial x^\mu}, \frac{\partial}{\partial x^\nu} \rangle = \frac{\partial x^a}{\partial x^\mu} \frac{\partial x^b}{\partial x^\nu} \langle \frac{\partial}{\partial x^a}, \frac{\partial}{\partial x^b} \rangle = \eta_{ab} e_\mu^a e_\nu^b$, and we may

have $g^{\mu\nu} = \langle dx^\mu, dx^\nu \rangle = \frac{dx^\mu}{dx^c} \frac{dx^\nu}{dx^d} \langle dx^c, dx^d \rangle = e_c^\mu e_d^\nu \eta^{cd}$. Thus this method is very convenient.

We still need to prove the consistence of indexes' raising and lowering by using covariant and contravariant metrics as follows

$$\begin{aligned} A^\mu &= g^{\mu\nu} A_\nu = \langle dx^\mu, dx^\nu \rangle A_\nu = \frac{dx^\mu}{dx^c} \frac{dx^\nu}{dx^d} \langle dx^c, dx^d \rangle A_\nu \\ &= e_c^\mu e_d^\nu \eta^{cd} A_\nu = e_c^\mu \eta^{cd} A_d = e_c^\mu A^c \end{aligned} \quad (\text{A5})$$

Analogously, we may prove $A_\mu = g_{\mu\nu} A^\nu = \langle \frac{\partial}{\partial x^\mu}, \frac{\partial}{\partial x^\nu} \rangle A^\nu = e_\mu^a A_a$. Thus, these studies are consistent.

Appendix B

Similar to deducing the important invariant $\omega_\mu \omega^\mu$ of inner product of two 1-forms, therefore, we deduce an important invariant of inner product of two 2-form Ω under coordinate transformation, as follows

$$\begin{aligned} \langle \langle \Omega, \Omega \rangle \rangle &= \langle \langle \frac{1}{2} \Omega_{\mu\nu} dx^\mu \wedge dx^\nu, \frac{1}{2} \Omega_{\mu'\nu'} dx^{\mu'} \wedge dx^{\nu'} \rangle \rangle \\ &= \frac{1}{4} \Omega_{\mu\nu} \Omega_{\mu'\nu'} g^{\nu\mu'} \langle dx^\mu, dx^{\nu'} \rangle = -\frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu}, \end{aligned} \quad (\text{B1})$$

substituting Eq.(5) into the deduced inner product expression (B1) and taking trace, then we have

$$tr \langle \langle \Omega_V, \Omega_V \rangle \rangle = -\frac{1}{4} tr(\Omega_{V\mu\nu} \Omega_V^{\mu\nu}) = tr \langle \langle \Omega_U, \Omega_U \rangle \rangle = -\frac{1}{4} tr(\Omega_{U\mu\nu} \Omega_U^{\mu\nu}), \quad (\text{B2})$$

which satisfies gauge invariant principle [21] of general gauge field's invariant, further using the invariant volume elements $(-g_U)^{\frac{1}{2}} d\tau_U = (-g_V)^{\frac{1}{2}} d\tau_V$ [19], we finally achieve the invariant action Eq.(10) of general gauge fields in the global curved spacetime.

References

- [1] G. Sterman, An Introduction to Quantum Field Theory, Cambridge University Press, New York 35 (1993).
- [2] L. Klein, Dispersion Relations and Abstract Approach to Field Theory, International Science Review Series, Vol. 1, Gordon and Breach Publishers Inc., New York 147 (1961).
- [3] P. Curie, Journal de physique, (Paris), 3rd series, 3(1894)395.
- [4] Yong-Chang (Y. C.) Huang , Mechanics Research Communications, 30 (2003) 567.
- [5] Y. C. Huang, X. G. Lee and M. X. Shao, Mod. Phys. Lett., A21(2006) 1107.

- [6] Y. C. Huang, F. C. Ma and N. Zhang, Mod. Phys. Lett., B18 (2005) 1367.
- [7] Y. C. Huang and B. L. Lin, Phys. Letts. A299 (2002) 644.
- [8] Y. C. Huang, C. Huang et al, International Journal of Theoretical Physics, 49 (2010) 2320-2333.
- [9] H. Weyl, Symmetry, Princeton University Press, Princeton, (1952).
- [10] E. Nöther, Nachr. Akad. Wiss., Gottingen, Math. Phys., Kl., II.235(1918).
- [11] D. S. Djukic, Int. J. Non-linear Mech., **8**(1993)479.
- [12] Z. P. Li, Int. J. Theor. Phys., **26**, 853(1987); Z. P. Li, J. H. Jiang, Symmetries in Constrained Canonical Systems, Science Press, New York, 2002.
- [13] K. Becker, M. Becker, John H. Schwarz, String Theory and M-Theory, Cambridge University Press, 2006.
- [14] Gary Shiu, Pablo Soler, and Fang Ye, Phys. Rev. Lett. 110, 241304 (2013); Gary Shiu, Bret Underwood, Kathryn M. Zurek, and Devin G. E. Walker, Phys. Rev. Lett. 100, 031601 (2008).
- [15] Mirjam Cvetič, Gary Shiu, and Angel M. Uranga, Phys. Rev. Lett. 87, 201801 (2001); Fernando Marchesano and Gary Shiu, Phys. Rev. D 71, 011701(R) (2005)
- [16] Novikov, Evgeny A., Ultralight gravitons with tiny electric dipole moment are seeping from the vacuum, MODERN PHYSICS LETTERS A,31 (2016) 1650092; Novikov, Evgeny A., Quantum modification of general relativity, Elect. J. Theoretical Physics, 13 (2016) 79-90.
- [17] S. S. Chern, Vector Bundles with a Connection, in Studies in Global Differential Geometry, Mathematical Association of America, 1989.
- [18] C. Nash and S. Sen, Topology and Geometry for Physicists, Academic Press, London 200 (1983).
- [19] B. Y. Hou etc, Differential Geometry for Physicists, World Scientific Publishing Co Pte Ltd, Singarbare (1997).
- [20] M. Carmeli, Classical Fields: General Relativity and Gauge Theory, A Wiley-Interscience Publication, John Wiley & Sons, New York 589 (1982).
- [21] X. S. Chen et al, Phys. Rev. Lett., 100 (2008) 232002.
- [22] D. Lurie, Particles and Fields, John Wiley & Sons Inc, Bristol 19 (1968).
- [23] Thomas P. Sotiriou and Valerio Faraoni, REVIEWS OF MODERN PHYSICS, 82, 2010, 451.
- [24] Review of Particle Physics, Phys. Rev. D 86 (2012) 010001.

- [25] M. E. Peskin and D. V. Schroeder, An Introduction to Quantum Field Theory (Beijing World Publishing Corp., 2006).
- [26] Y. C. Huang and C. X. Yu, Phys. Rev. D 75 (2007)044011; J. C. Hua and Y. C. Huang, Europhysics Letters, 85 (2009) 30007; Y. C. Huang and Q. H. Huo, Physics Letters, B662(2008) 290-296.
- [27] Y. C. Huang, L. Liao and X. G. Lee, The European Physical Journal, C60 (2009) 481-487; L. Liao and Y. C. Huang, Phys. Rev. D 75(2007)025025; Y. C. Huang and L. X. Yi, Annals of Physics (New York), 325 (2010) 2140-2152; B. H. Zhou, and Y. C. Huang, Phys. Rev. D 84, 047701 (2011).
- [28] M. Wasay Abdul, Y. C. Huang, D. F. Zeng, Quantization and spectrum of RNS supersymmetric open 2-brane, Nuclear Physics, B892 (2015) 353-363.
- [29] G. R. Chen, Y. C. Huang, Recovering information of tunneling spectrum from Weakly Isolated Horizon, European Physical Journal, C75 (2015)47.